

Influence of 3D-VAR Dissimilarity Measure on Flood Model Calibration with Satellite Data

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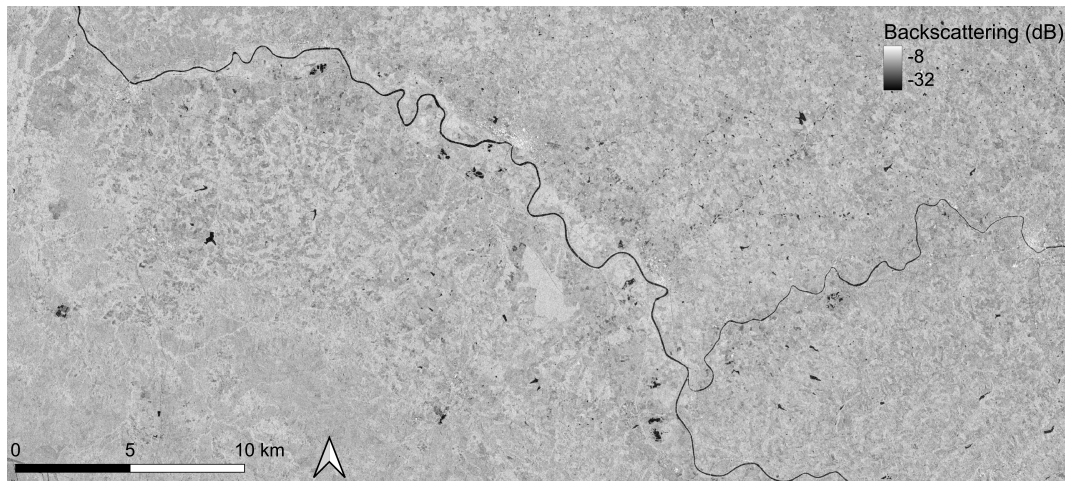


Outline

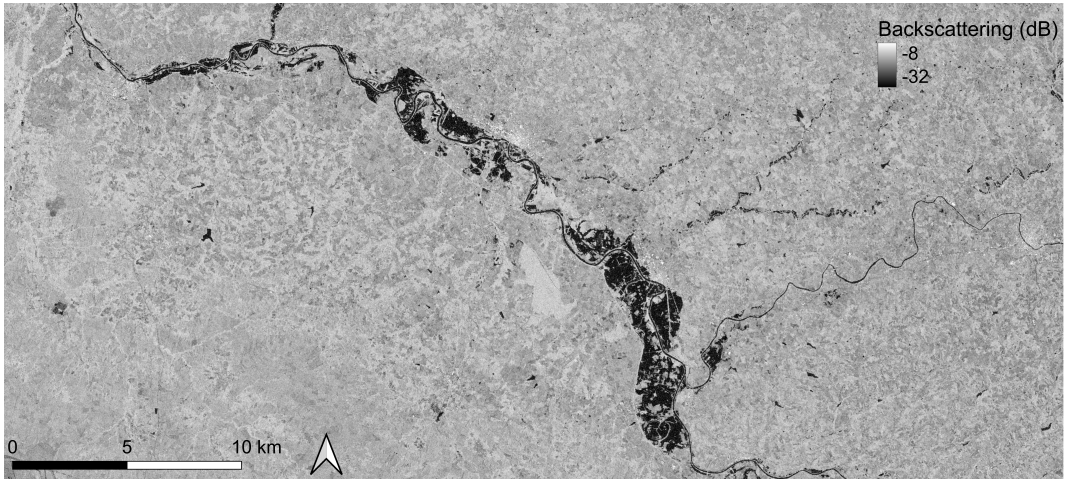
- 1 Context
- 2 Data-driven parameter estimation
- 3 Test case
- 4 Numerical study
- 5 Conclusions and perspectives



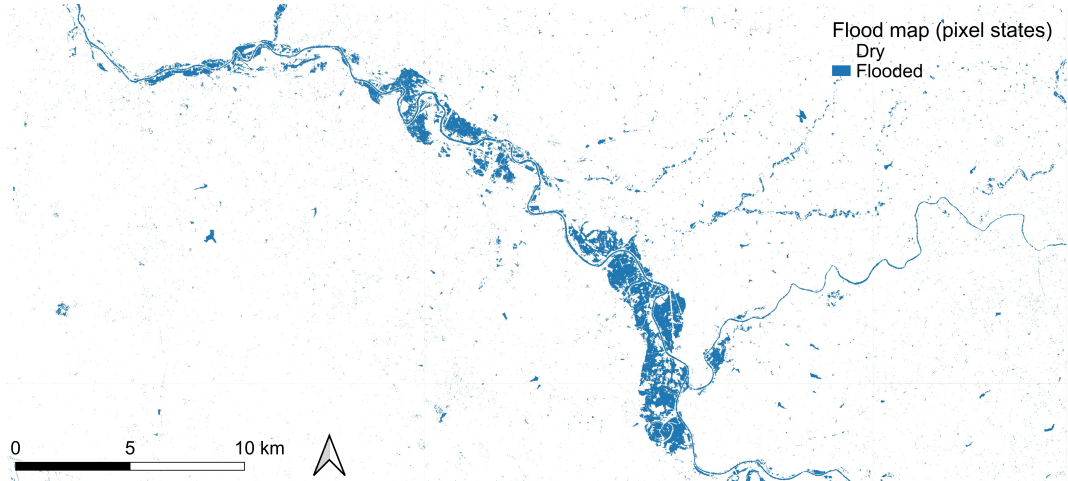
Example of flood detection: Garonne River in France



Example of flood detection: Garonne River in France



Example of flood detection: Garonne River in France



Friction coefficients in flood models

Context

- Modeling floods on floodplains
- Shallow-Water Equations modeling:

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = 0 \\ \frac{\partial(hu)}{\partial t} + \frac{\partial(hu^2)}{\partial x} + \frac{\partial(huv)}{\partial y} = -gh \frac{\partial \eta}{\partial x} - \frac{1}{\rho} \tau_{bx}(K_s, h, u) + \nabla \cdot (h v_e \nabla u) \\ \frac{\partial(hv)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial(hv^2)}{\partial y} = -gh \frac{\partial \eta}{\partial y} - \frac{1}{\rho} \tau_{by}(K_s, h, v) + \nabla \cdot (h v_e \nabla v) \end{cases}$$

- Role of friction coefficients $K_s(x, y)$ on hydraulics variables

Objective

- Data-driven quantification of friction coefficients on floodplains
 - Use satellite flood maps to calibrate a flood model

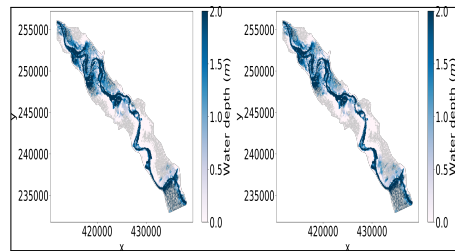
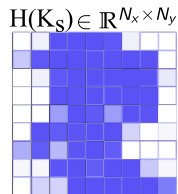


Figure 1: Illustration of the influence of $K_s(x, y)$ parametrization on water depths for two configurations.

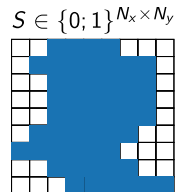
Impact of measure on calibration

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = 0 \\ \frac{\partial(hu)}{\partial t} + \frac{\partial(hu^2)}{\partial x} + \frac{\partial(huv)}{\partial y} = -g h \frac{\partial \eta}{\partial x} - \frac{1}{\rho} \tau_{bx}(K_S, h, u) + \nabla \cdot (h \mathbf{v}_e \nabla u) \\ \frac{\partial(hv)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial(hv^2)}{\partial y} = -g h \frac{\partial \eta}{\partial y} - \frac{1}{\rho} \tau_{by}(K_S, h, v) + \nabla \cdot (h \mathbf{v}_e \nabla v) \end{cases}$$

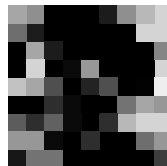
At fixed T
→
Projected
on grid



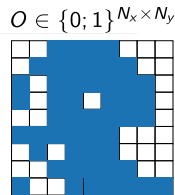
→
 $1_{H_{i,j} > \tau}$



SAR
observation



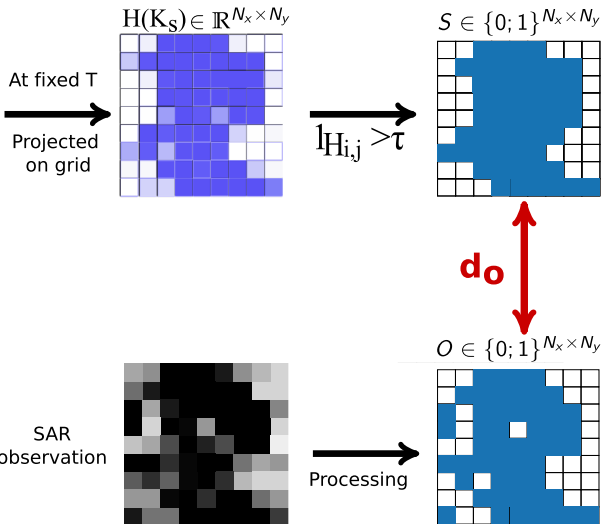
→
Processing



Impact of measure on calibration

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Influence of d_o on K_S calibration?



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- 1 Context
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Notations

- ▶ $\mathcal{M}$ flood model to calibrate
- ▶ $\theta \in \mathbb{R}^d$ parameters to calibrate
- ▶ $\theta^b \in \mathbb{R}^d$ prior estimate of the parameters
- ▶ $\theta^t \in \mathbb{R}^d$ true values of the parameters
- ▶ $\mathbf{y} = O \in \{0; 1\}^{N_x \times N_y}$ flood maps
- ▶ $\mathcal{H}$ an operator from state to observation space

Automatic calibration

Classical 3D-VAR

- ▶ Minimise a cost function:

$$\begin{cases} J(\boldsymbol{\theta}) = J_b(\boldsymbol{\theta}) + J_o(\boldsymbol{\theta}) \\ J_b(\boldsymbol{\theta}) = \frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\theta}^b)^T \mathbf{B}^{-1}(\boldsymbol{\theta} - \boldsymbol{\theta}^b) \\ J_o(\boldsymbol{\theta}) = \frac{1}{2}(\mathbf{y} - \mathcal{H}(\mathcal{M}(\boldsymbol{\theta})))^T \mathbf{R}^{-1}(\mathbf{y} - \mathcal{H}(\mathcal{M}(\boldsymbol{\theta}))) \end{cases},$$

where $\mathcal{H}(\mathcal{M}(\boldsymbol{\theta})) = S(\boldsymbol{\theta})$

- ▶ Gradient descent with Broyden-Fletcher-Goldbarb-Shanno
- ▶ Implemented with ADAO (data assimilation and optimisation) and TelApy

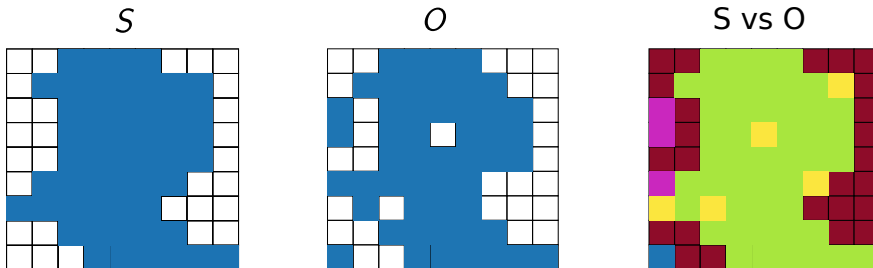
Automatic calibration

Generalized cost function

$$J(\boldsymbol{\theta}) = \omega_b d_b(\boldsymbol{\theta}, \boldsymbol{\theta}^b)^2 + d_o(\mathbf{y}, \mathcal{H}(\mathcal{M}(\boldsymbol{\theta})))^2$$

- ▶ $d_b(\boldsymbol{\theta}, \boldsymbol{\theta}^b) = \sqrt{\sum_{i=1}^d (\theta_i - \theta_i^b)^2}$ (Usual Euclidean distance)
- ▶ d_o a dissimilarity measure to specify
- ▶ ω_b (set to 10^{-12}) a weight balancing the background knowledge and the observation/model misfit

Dissimilarity measures: Pixel to pixel



	OBSERVED FLOODED	OBSERVED DRY
SIMULATED FLOODED	<i>TP</i>	<i>FP</i>
SIMULATED DRY	<i>FN</i>	<i>TN</i>

Usual Euclidean distance

$$d_e(O, S) = \sqrt{\sum_{i=1}^{N_x} \sum_{j=1}^{N_y} (O_{i,j} - S_{i,j})^2} = \sqrt{FN + FP}$$

Measure of fit

$$F^4(O, S) = \frac{TP - FP}{TP + FP + FN}$$

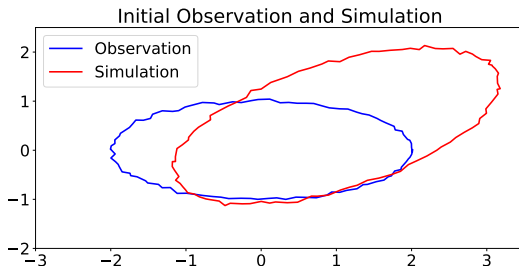
Dissimilarity measures (2)

Procrustes analysis

- Translations, Rotations, and Scaling of O and S to minimise:

$$P^2 = d_e(O - cSQ),$$

where c is a scaling factor, and Q an orthogonal matrix



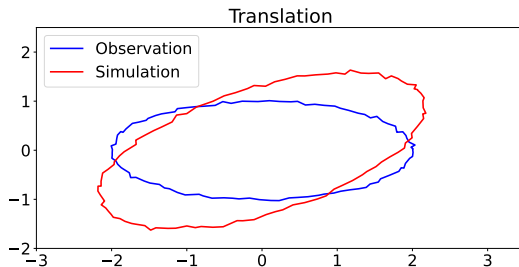
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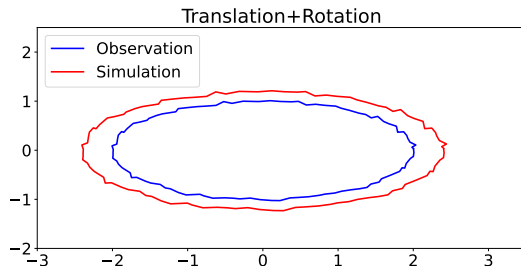
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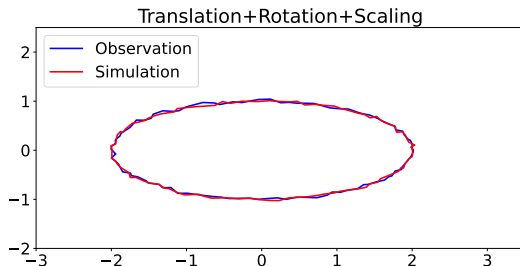
Dissimilarity measures (2)

Procrustes analysis

- Translations, Rotations, and Scaling of O and S to minimise:

$$P^2 = d_e(O - cSQ),$$

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Dissimilarity measures (3)

Normalized Mutual Information

Based on information theory

$$NMI(O, S) = \frac{H(O) + H(S)}{H(O, S)},$$

where $H(O) = -\sum_{i=1}^{nc} p_i \log(p_i)$ with nc the number of classes (2 here with flooded and dry pixels) and p_i the probability of class i .

$H(O, S) = -\sum_{i=1}^{nc} \sum_{j=1}^{nc} p_{i,j} \log(p_{i,j})$ with $p_{i,j}$ the joint probability between classes i in O and j in S .

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Schematic test case

- ▶ $\mathcal{M}$: Shallow-Water Equations (TELEMAC-2D) **openTELEMAC**
- ▶ Twin experiment: Overflow of a meandering channel with $\mathcal{N} = 7\,979$ nodes
- ▶ Boundary conditions: Rating curve at the outlet and varying discharge at the inlet

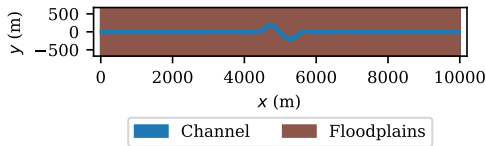


Figure 2: Schematic meandering river with floodplains.

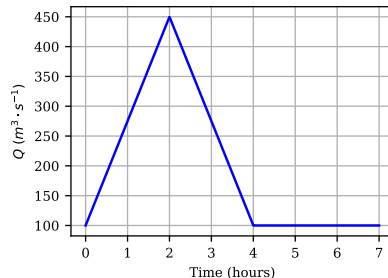


Figure 3: Upstream flood hydrograph during the simulation.

Schematic test case: Control vector

- ▶ Control vector $\theta = [K_{s1}, K_{s2}, K_{s3}]$ with observations generated for $\theta^t = [30; 20; 10]$

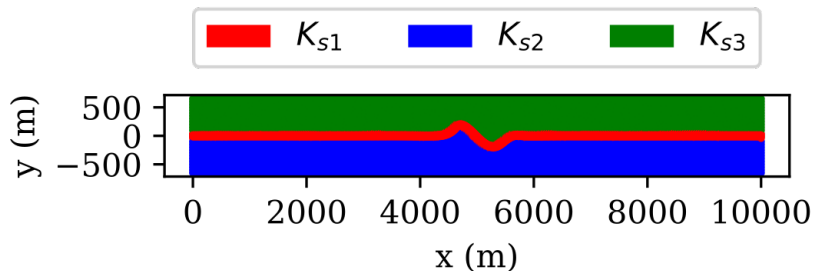


Figure 4: Schematic case with 3 zones to calibrate.

- ▶ One temporal observation at $T = 3 \text{ hours}$

Twin experiment: $\mathcal{H}$ operator

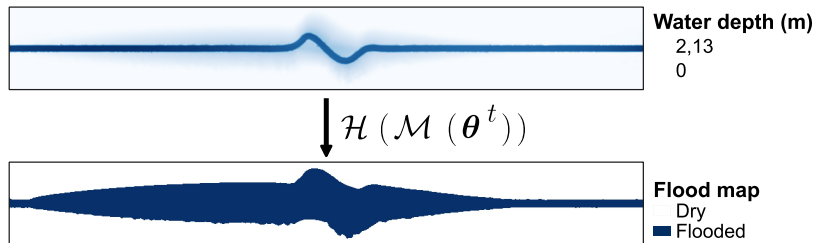


Figure 5: Observation O generated with $\theta^t = [30; 20; 10]$ at 3 hours of simulation.

$$[\mathcal{H}(\mathcal{M}(\theta))]_{i,j} = S_{i,j}(\theta) = \begin{cases} 1 & \text{if } H_{i,j} > 0.01 \text{ m} \\ 0 & \text{otherwise} \end{cases}$$

- Calibration framework $\Rightarrow$ without noise in the observations

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Numerical considerations

3D-VAR parametrization

- ▶ Bounds of parameter estimation: $20 \leq K_{s1} \leq 40$, and $5 \leq K_{s2}, K_{s3} \leq 50$
- ▶ Initial guess $\theta^b = [35, 12, 18]$
- ▶ End of the algorithm: 35 iterations or the cost function decreases less than 10^{-4}

Influence of d_o on parameter estimation

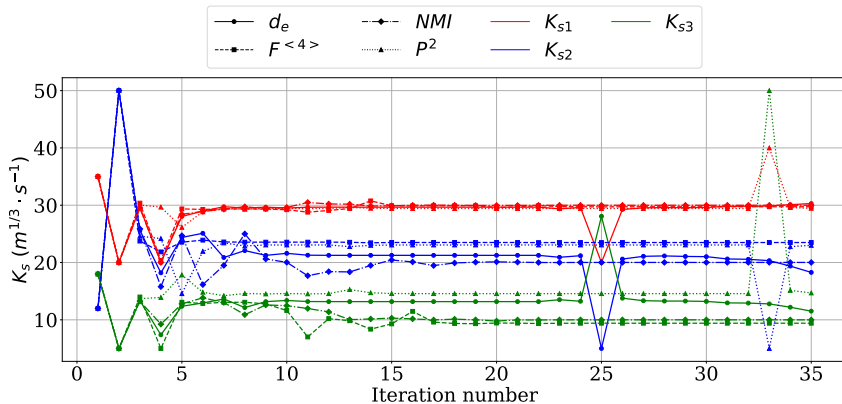
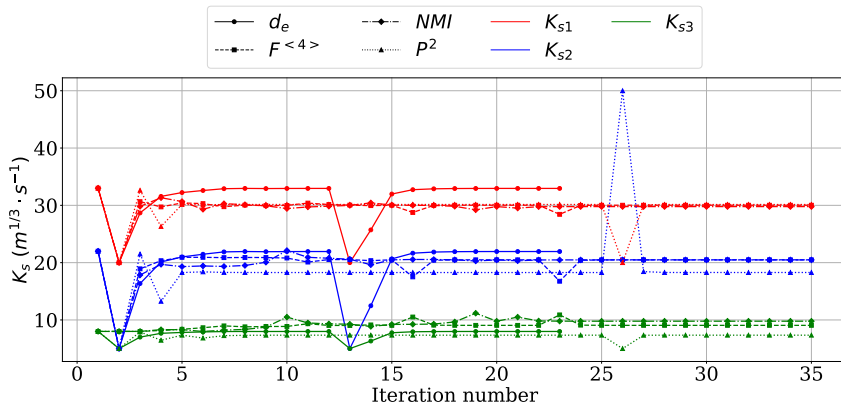


Figure 6: Automatic calibration of θ for the four dissimilarity measures (circles for d_e (Euclidean distance), diamonds for NMI (Normalised Mutual Information), squares for $F^{<4>}$ (Measure of fit), and triangles for P^2 (Procrustes analysis)) for $\theta^b = [35; 12; 18]$.

► NMI retrieves θ^t for this initial θ^b : Robustness of identification to θ^b ?

Influence of θ^b on parameter estimation

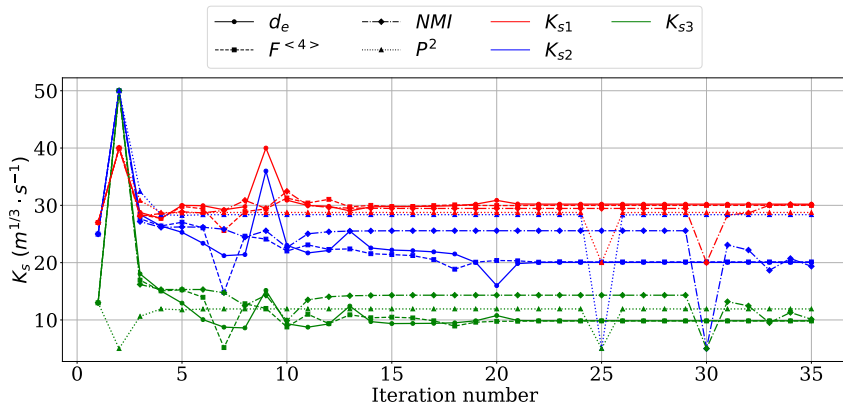
► $\theta^b = [33, 22, 8]$



► $F^{<4>}$ converges to θ^t while it was not the case with previous initialization

Influence of θ^b on parameter estimation

► $\theta^b = [27, 25, 13]$



► NMI requires longer to converge, and P^2 still fails to converge to θ^t

d_o coupled variations

From some initial states the behavior differs $\Rightarrow$ Smoothness of $\mathcal{H}$ or convexity issues?

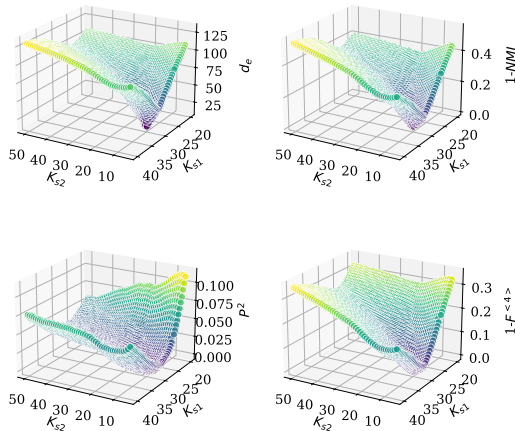


Figure 7: Variation of the four dissimilarity measures as a function of the coupled parameters K_{s1} and K_{s2} for $K_{s3} = 10$.

d_o coupled variations

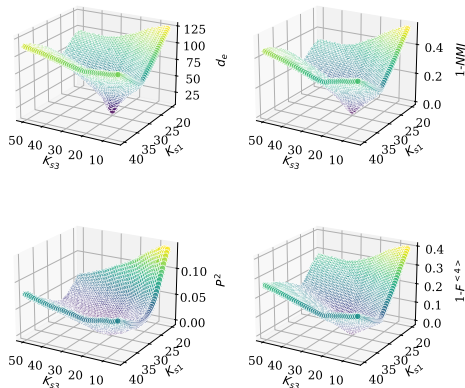


Figure 8: Variation of the four dissimilarity measures as a function of the coupled parameters K_{s1} and K_{s3} for $K_{s2} = 20$.

- P^2 and $F^{<4>}$ have smaller gradients for variations of channel or floodplains friction

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Conclusions and perspectives

- ▶ Automatic calibration of TELEMAC-2D parameters with variational data assimilation algorithms
- ▶ Importance of choosing an adapted dissimilarity measure for parameter estimation
- ▶ Normalized Mutual Information was the most robust and converged faster on this test case
- ▶ Impact of the initial state
- ▶ Need to add noise on the observations, and apply the methodology to real flood events

Thank you for your attention!